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Anomalous Generation of Chern–Simons Terms in $D = 3$, $N = 2$ Supersymmetric Gauge Theories

E. R. NISSIMOV and S. J. PACHEVA
Institute for Nuclear Research and Nuclear Energy, Boul. Lenin 72, 1784 Sofia, Bulgaria

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Abstract. Parity-invariant three-dimensional gauge theories with $N = 2$ extended supersymmetry are studied by the heat kernel method. The parity-anomalous part of the one-loop effective action is exactly found. It is expressed in terms of the $N = 2$ supersymmetric Chern–Simons term and is identified as a $N = 2$ superspace Atiyah–Patodi–Singer eta-invariant.

1. Recently, anomalous violation of parity (space-reflection symmetry) in parity-invariant gauge theories with fermions in odd (Euclidean) spacetime dimensions D was observed [1–3] and extensively investigated [4–8]. The interest in these parity-violating anomalies (PVA) arises due to the fact that PVA are direct counterparts of the well-known chiral anomalies in even D [9] as well as due to their relevance for some interesting physical phenomena like fermion number fractionization [3, 4] and the Hall effect [5].

Parity-violating anomalies in the one-loop effective action appear because of the spectral asymmetry for odd D of the Dirac operator $\not\nabla(A)$ in external background gauge field. In particular, the usual squaring ('doubling' trick) $\ln\det[\not\nabla(A)] = \frac{1}{2}\ln\det[\not\nabla^2(A)]$ is, in general, not correct for odd D.

In [10], the analysis of PVA was extended to $D = 3$ gauge theories with $N = 1$ supersymmetry. The parity-violating part of the one-loop effective action was exactly computed using the superspace heat kernel method [11] and it was identified as the $N = 1$ superspace Atiyah–Patodi–Singer η-invariant [12] of the operator appearing in the bilinear in the scalar superfields part of the classical action. This identification was made because, in analogy with the ordinary case, the $N = 1$ superspace η-invariant equalled twice the $N = 1$ supersymmetric Chern–Simons term plus a piecewise constant (even-integer valued) functional.

The aim of the present Letter is to study PVA in parity-invariant extended ($N = 2$) supersymmetric gauge theories in $D = 3$ by the heat kernel method. In analogy with the ordinary and $N = 1$ cases, the $N = 2$ PVA are obtained in terms of twice the $N = 2$ supersymmetric Chern–Simons term plus a piecewise constant functional, which once again is identified as a $N = 2$ superspace η-invariant (Equation (20) below). In particular, the 'doubling' trick in the superspace background-field method [13] is not, in general, valid in $D = 3$. Also, general conditions for cancellation of PVA are found.

2. The classical action of the $N = 2$ supersymmetric model under consideration reads:

$$S = -\int \mathrm{d}^3x\, \mathrm{d}^4\theta\, \overline{\Phi}\, \mathrm{e}^V \Phi\,. \tag{1}$$

It is obtained by dimensional reduction from the $D = 4$, $N = 1$ supersymmetric model of chiral scalar superfields $\Phi = (\Phi^i_a)$ ($i = 1, \dots, n$; $a = 1, \dots, N_f$ – number of 'flavors') belonging to the fundamental representation of $\mathrm{U}(n)_{\text{gauge}} \times \mathrm{U}(N_f)$ and interacting with a background $\mathrm{U}(n)$ gauge prepotential $V = (V^{ij})$. In terms of component-fields★, where the Wess–Zumino gauge is imposed on V, the action (1) takes the form:

$$S' = \int \mathrm{d}^3x \{\tfrac{1}{2}\varphi^*[-\nabla^2(A) - A_0^2 + D']\varphi + F^*F + \overline{\psi}[\nabla\!\!\!/(A) + A_0]\psi + $$
$$+ i\overline{\psi}(\overline{\lambda}C)\varphi - i\varphi^*(C\lambda)\psi\}\,; \tag{2}$$

$$\nabla^2(A) = \nabla_m(A)\nabla_m(A)\,, \qquad \nabla_m(A) = \partial_m + iA_m(x)\,, \qquad \nabla\!\!\!/(A) = \gamma_m\nabla_m(A)\,,$$

$\{\gamma_m, \gamma_n\} = -2\delta_{mn}$, $m, n = 1, 2, 3$; $\gamma_m = i\sigma_m$, $C = \sigma_2$ (σ_m denotes Pauli matrices); $\overline{\psi}_{(1)}\psi_{(2)} = \Sigma_{\alpha=1,2}\, \overline{\psi}^{(\alpha)}_{(1)}\psi^{\alpha}_{(2)}$, etc., for any pair of spinors.

Summation over repeated indices is understood and the latter will be suppressed for brevity.

The action (2) is invariant under $\mathrm{U}(n)$ gauge (Wess–Zumino gauge-preserving) supersymmetry and parity (space-reflection) transformations (we need them explicitly only for the V-components):

$$A^g_m = g^{-1}(A_m - i\,\partial_m)g\,, \qquad A^g_0 = g^{-1}A_0 g \quad \text{(the same for } D', \lambda, \overline{\lambda}\text{)}$$
$$g: \mathbb{R}^3 \to G = \mathrm{U}(n)\,, \qquad g(x) \xrightarrow[|x|\to\infty]{} \mathbb{1}\,; \tag{3}$$

$$\delta A_0 = \frac{i}{2}\,(\overline{\lambda}\varepsilon - \overline{\varepsilon}\lambda)\,, \qquad \delta A_m = \tfrac{1}{2}(\overline{\lambda}\gamma_m\varepsilon - \overline{\varepsilon}\gamma_m\lambda)\,,$$

$$\delta D' = -\frac{i}{2}\,[A_0, \overline{\varepsilon}\lambda + \overline{\lambda}\varepsilon] - \frac{i}{2}\,(\overline{\varepsilon}\nabla\!\!\!/(A)\lambda + (\overline{\lambda}\overleftarrow{\nabla\!\!\!/}(A))\varepsilon)\,, \tag{4}$$

$$\delta\lambda = (i\nabla\!\!\!/(A)A_0 + \tfrac{1}{2}\gamma_m{}^*F_m(A) + iD')\varepsilon\,,$$

where $\nabla_m(A)\lambda = \partial_m\lambda + i[A_m, \lambda]$ (analogously for $\nabla_m(A)A_0$),

$${}^*F_m(A) = \tfrac{1}{2}\varepsilon_{mnl}F_{nl}(A)\,, \qquad F_{mn}(A) = \partial_m A_n - \partial_n A_m + i[A_m, A_n]\,;$$
$$A^P_0(x) = -A_0(x_P)\,, \qquad A^P_m(x) = (-A_1, A_2, A_3)(x_P)\,, \qquad D'^P(x) = D'(x_P)\,, \tag{5}$$
$$\lambda^P(x) = i\gamma_1\lambda(x_P)\,, \qquad x_P \equiv (-x^1, x^2, x^3)\,.$$

In the sequel, the following standard boundary conditions for the component-fields

★ In notations and notions we essentially follow the book [13].

of V will be assumed:

$$A_m(x) \xrightarrow[|x|\to\infty]{} -ih^{-1}(x)\,(\partial_m h)\,(x) + \mathcal{O}(|x|^{-1-\delta}), \quad h(x)\in \mathrm{U}(n),$$

$$\lambda(x), \bar{\lambda}(x), D'(x) = \mathcal{O}(|x|^{-1-\delta}), \quad \text{for } |x| \to \infty . \tag{6}$$

3. The one-loop quantum effective action S_{eff} is directly computed (the corresponding functional integral is Gaussian):

$$\exp(-N_f S_{\text{eff}}[A_m, A_0, \lambda, \bar{\lambda}, D']) = \int \mathcal{D}\varphi\mathcal{D}\varphi^*\mathcal{D}\psi\mathcal{D}\bar{\psi}\mathcal{D}F\mathcal{D}F^*\,\mathrm{e}^{-S'},$$

$$S_{\text{eff}} = \operatorname{Tr}\ln[-\nabla^2(A) - A_0^2 + D'] -$$
$$- \operatorname{Tr}\ln[\not\nabla + A_0 - (\bar{\lambda}C)(-\nabla^2(A) - A_0^2 + D')^{-1}(C\lambda)] . \tag{7}$$

Now, in order to regularize the logarithms of the functional determinants appearing in (7), we choose the method first proposed by Polyakov [14] (see also [6]). For any elliptic (self-adjoint) operator Q on $\mathbb{R}^D$ one defines:

$$(\operatorname{Tr}\ln Q)^{\text{reg}} = \tfrac{1}{2}(\operatorname{Tr}\ln Q^2)^{\text{reg}} - i\,\frac{\pi}{2}\,\eta_Q , \tag{8}$$

$$(\operatorname{Tr}\ln Q^2)^{\text{reg}} = -\int_0^\infty \frac{\mathrm{d}\tau}{\tau}\operatorname{Tr}_R[\mathrm{e}^{-\tau Q^2}] = -\frac{\mathrm{d}}{\mathrm{d}s}\,\zeta_{Q^2}(s)\Big|_{s=0} , \tag{9}$$

$$\eta_Q = \int_0^\infty \mathrm{d}\lambda\,\operatorname{sign}(\lambda)\operatorname{Tr}_R[\mathcal{P}_Q(\lambda)] = \int_0^\infty \mathrm{d}\tau(\pi\tau)^{-1/2}\operatorname{Tr}_R[Q\,\mathrm{e}^{-\tau Q^2}] . \tag{10}$$

Equation (9) represents the ζ-function regularization of ln det of the nonnegative operator Q^2 [15]. η_Q (10) denotes the well-known spectral asymmetry measuring η-invariant [12] of $Q = \int \mathrm{d}\lambda\,\mathcal{P}_Q(\lambda)\lambda$, where $\mathcal{P}_Q(\lambda)$ is the spectral density of Q. In (9) and (10) $\operatorname{Tr}_R[\ldots] = \operatorname{Tr}[\chi_R(\ldots)]$ means an infrared regularized operator trace, where χ_R denotes the multiplication operator by the characteristic function of a compact subset $K_R \subset \mathbb{R}^D$. At the end of computations one takes $K_R \to \mathbb{R}^D$. In the case of boundary conditions (6), no boundary terms arise in this limit.

Application of formula (8) to (7) yields the renormalized S_{eff}:

$$S_{\text{eff}}^{\text{ren}} = \tfrac{1}{2}(\operatorname{Tr}\ln[\Delta_B^2])^{\text{reg}} - \tfrac{1}{2}(\operatorname{Tr}\ln[\Delta_F^2])^{\text{reg}} - i\,\frac{\pi}{2}\,(\eta_{\Delta_B} - \eta_{\Delta_F}) + S_{\text{c.t.}} , \tag{11}$$

$$\Delta_B \equiv -\nabla^2(A) - A_0^2 + D' , \qquad \Delta_F \equiv \not\nabla + A_0 - (\bar{\lambda}C)\Delta_B^{-1}(C\lambda) , \tag{12}$$

where $S_{\text{c.t.}} = S_{\text{c.t.}}[A_m, A_0, \lambda, \bar{\lambda}, D']$ denotes the finite counterterm action accounting for the renormalization ambiguity. $S_{\text{c.t.}}$ should satisfy the following requirements:

(a) It must be a local $\mathrm{U}(n)$ gauge-invariant functional of $A_m, A_0, \lambda, \bar{\lambda}, D'$, of dimension $D = 3$;

(b) It must be chosen in such a way as to eventually cancel the supersymmetry breaking and PVA terms coming from η_{Δ_F} in (11).

Indeed, under parity-transformation (5) we have:

$$\Delta_F[A_m^P, A_0^P, \lambda^P, \bar{\lambda}^P, D'^P](x, x') = \gamma_1 \Delta_F[A_m, A_0, \lambda, \bar{\lambda}, D'](x_P, x'_P)\gamma_1 ,$$

and, therefore, accounting for (11) (and recalling $\gamma_1^2 = -1$):

$$\eta_{\Delta_F}[A_m^P, \ldots, D'^P] = -\eta_{\Delta_F}[A_m, \ldots, D'] . \tag{13}$$

Thus, the appearance of η_{Δ_F} in (11) represents the general form of PVA unless the PVA could be cancelled by an appropriate $S_{\text{c.t.}}$ (see below).

Also, it can be directly verified using (4) that η_{Δ_F} breaks $N = 2$ supersymmetry (cf. (15b) below).

4. The exact computation of η_Q for any Q proceeds as follows. Starting from the 'proper-time' expression (10), the variation of η_Q under variation of the coefficients of Q yields [6, 8]:

$$\delta\eta_Q = 2\pi^{-1/2} \int_0^\infty \mathrm{d}\alpha \, \frac{\mathrm{d}}{\mathrm{d}\alpha} \left(\mathrm{Tr}[(\delta Q)\exp(-\alpha^2 Q^2)]\right)$$

$$= 2\,\mathrm{Tr}[(\delta Q)\mathscr{P}_Q(0)] - 2\pi^{-1/2} \int \mathrm{d}^D x \, \mathrm{tr}[\Phi^{(D)}_{-1/2}(Q^{(2)}; \delta Q \,|\, x)] ; \tag{14a}$$

$$\eta_Q = \eta_Q^{\text{disc}} + \eta_Q^{\text{cont}} ; \tag{14b}$$

where the asymptotic formula

$$\exp(-\tau Q^2) \xrightarrow[\tau \to \infty]{} \left(\frac{\pi}{\tau}\right)^{1/2} \mathscr{P}_Q(0)$$

was used and $\mathrm{tr}[\ldots]$ means trace over discrete indices. The η_Q^{disc} in (14b) is a piecewise constant functional of the coefficients of Q taking even-integer values (it represents the contribution of the jumps of η_Q by ± 2 whenever Q acquires zero modes [12]). The continuous part in (14b) η_Q^{cont} is a local functional of the coefficients of Q. The variation of η_Q^{cont} (the last term in (14a)) is expressed through the coefficient $\Phi^{(D)}_{-1/2}(Q^2; \delta Q \,|\, x)$ in the asymptotic Seeley expansion [16]:

$$[\delta Q \, \mathrm{e}^{-\tau Q^2}](x, x) = \sum_{j=0}^{\infty} \tau^{(j-s-D)/2r} \, \Phi^{(D)}_{(j-s-D)/2r}(Q^{(2)}; \delta Q \,|\, x)$$

where $2r$, s are orders of the principal (highest derivative) parts of Q^2, δQ, respectively.

The symbol calculus of pseudo-differential operators [17] provides an efficient tool for a straightforward computation of the Seeley coefficients. Application of this formalism in the case of $Q = \Delta_B, \Delta_F$ gives the following results:

$$\eta_{\Delta_B} = \eta_{\Delta_B}^{\text{disc}} \, (\eta_{\Delta_B}^{\text{cont}} = 0) , \tag{15a}$$

$$\eta_{\Delta_F} = \eta_{\Delta_F}^{\text{disc}} + 2W^{(3)}_{\text{ChS}}[A] + (4\pi^2)^{-1} \int \mathrm{d}^3x \, [\operatorname{tr}(\bar{\lambda}\lambda) + \tfrac{1}{6}\operatorname{tr}(A_0^3)] \,, \tag{15b}$$

where $W^{(3)}_{\text{ChS}}[A]$ denotes the well-known $D = 3$ Chern–Simons term – the topological mass term for $A_m(x)$ [13, 18].

$$W^{(3)}_{\text{ChS}}[A] = -(16\pi^2)^{-1} \varepsilon_{mnl} \int \mathrm{d}^3x \operatorname{tr}[A_m F_{nl}(A) - i\tfrac{2}{3}A_m A_n A_l] \,. \tag{16}$$

Let us recall that under gauge (3) and parity (5) transformations:

$$W^{(3)}_{\text{ChS}}[A^g] = W^{(3)}_{\text{ChS}}[A] + N^{(3)}[g] \,, \qquad W^{(3)}_{\text{ChS}}[A^P] = -W^{(3)}_{\text{ChS}}[A] \,; \tag{17}$$

$$N^{(3)}[g] = -(24\pi^2)^{-1} \varepsilon_{mnl} \int \mathrm{d}^3x \operatorname{tr}[(g^{-1}\partial_m g)(g^{-1}\partial_n g)(g^{-1}\partial_l g)] \,, \tag{18}$$

$\mathrm{N}^{(3)}[\mathrm{g}] \in \mathbb{Z}$ for $G = \mathrm{U}(n)$, $n \geqslant 2$, $N^{(3)}[g] \equiv 0$ for $G = \mathrm{U}(1)$, where $N^{(3)}[g]$ (element of $\pi_3(\mathrm{U}(n))$) is the topological charge of $g(x)$.

From (4), (5), and (16) one directly checks that, first, η_{Δ_F} (15b) is indeed parity-odd (13) and, second, (15b) is not invariant under $N = 2$ supersymmetry. The latter may be easily remedied by adding the following finite local counterterm (cf. (11)):

$$S^{(1)}_{\text{c.t.}} = -i(4\pi^2)^{-1} \int \mathrm{d}^3x \operatorname{tr}[A_0 D' + \tfrac{1}{6}A_0^3] \,;$$

$$\eta_{\Delta_F}^{\text{SuSy}} \equiv \eta_{\Delta_F} - i\,\frac{2}{\pi}\, S^{(1)}_{\text{c.t.}} = \eta_{\Delta_F}^{\text{disc}} + \\ + 2W^{(3)}_{\text{ChS}}[A] + (4\pi^2)^{-1} \int \mathrm{d}^3x \operatorname{tr}[\bar{\lambda}\lambda - 2A_0 D'] \,. \tag{19}$$

Now, the last line of Equation (19) is immediately recognized as twice the component-field form (in the Wess–Zumino gauge) of the $D = 3$, $N = 2$ supersymmetric Chern–Simons term $W^{\text{SuSy}}_{\text{ChS}}[V]$ [19]. Moreover, accounting for (11) and (8) the quantity:

$$\eta_H^{\text{SuSy}}[V] \equiv \eta_{\Delta_B} - \eta_{\Delta_F}^{\text{SuSy}} = \eta_H^{\text{disc}}[V] - 2W^{\text{SuSy}}_{\text{ChS}}[V] \,, \\ (\eta_H^{\text{disc}}[V] \equiv \eta_{\Delta_B}^{\text{disc}} - \eta_{\Delta_F}^{\text{disc}}) \tag{20}$$

should be identified as $N = 2$ superspace η-invariant of the $N = 2$ superspace differential operator H entering the action (1):

$$H = \begin{pmatrix} 0 & \bar{D}^2 \\ D^2 \mathrm{e}^V & 0 \end{pmatrix}, \qquad \overleftrightarrow{D}^2 \equiv -\tfrac{1}{2}\overleftrightarrow{D}C\overleftrightarrow{D} \,, \tag{21}$$

(Equation (21) is the form explored in [11]; D_α, $\bar{D}_\alpha$ being the standard spinor superderivatives [13]). Also, let us note that in the Wess–Zumino gauge

$$\ln\det[H^2] = \operatorname{Tr}\ln[\Delta_B^2] - \operatorname{Tr}\ln[\Delta_F^2] \,. \tag{22}$$

Therefore, combining (20)–(22), (11) may be recast into a $N = 2$ supersymmetric form:

$$\begin{aligned} N_f S_{\text{eff}}^{\text{ren}}[V] &= N_f \ln \det[H(V)] \\ &= \tfrac{1}{2} N_f \ln \det[H^2(V)] - i\,\frac{\pi}{2}\, N_f \eta_H^{\text{SuSy}}[V] + N_f S_{\text{c.t.}}[V] \\ &= \tfrac{1}{2} N_f \ln \det[H^2(V)] - i\,\frac{\pi}{2}\, N_f \eta_H^{\text{disc}}[V] + \\ &\quad + i\pi N_f W_{\text{ChS}}^{\text{SuSy}}[V] + N_f S_{\text{c.t.}}[V]\,, \end{aligned} \tag{23}$$

where $\eta_H^{\text{SuSy}}[V]$ (20) represents the PVA and the number of 'flavors' N_f is explicitly indicated. Let us emphasize the complete analogy of the $N = 2$ superspace formula (23) with the corresponding one for ordinary operators (8).

5. Finally, let us discuss the possibilities of cancelling PVA in (23) through finite local $S_{\text{c.t.}}[V]$. From (20) it is clear that the whole nonlocality of the PVA terms is contained in the (even-integer)-valued $\eta_H^{\text{disc}}[V]$ which, according to (19) and (17), changes under gauge transformations (3) as:

$$\eta_H^{\text{disc}}[A_m^g, \ldots, D'^g] = \eta_H^{\text{disc}}[A_m, \ldots, D'] - 2N^{(3)}[g] \tag{24}$$

(note that η_{Δ_B}, η_{Δ_F}, η_H^{SuSy} are gauge-invariant). Thus, according to (24) and (23), we arrive at the following two alternatives (which parallel those in the ordinary [1, 8] and $N = 1$ supersymmetric cases [10]):

(i) If either $G = \mathrm{U}(1)$ or if N_f = even for $G = \mathrm{U}(n)$, $n \geqslant 2$, we can choose:

$$S_{\text{c.t.}}^{(2)}[V] = -i\pi W_{\text{ChS}}^{\text{SuSy}}[V]\,, \tag{25}$$

$$N_f S_{\text{eff}}^{\text{ren}}[V] = \tfrac{1}{2} N_f \ln \det[H^2(V)] - i\,\frac{\pi}{2}\, N_f \eta_H^{\text{disc}}[V]\,, \tag{26}$$

and since in this case $\frac{1}{2}\pi N_f \eta_H^{\text{disc}}[V] = 0 \pmod{2\pi}$ the PVA in (23) are eliminated through (25).

(ii) If $G = \mathrm{SU}(n)$, $n \geqslant 2$, and N_f = odd, simultaneously, then the choice of (25) and (26), while eliminating PVA, breaks gauge invariance (cf. (24)). Therefore, in this case, the PVA are unavoidable. In particular, the 'doubling' trick [13] $\ln \det[H] = \frac{1}{2} \ln \det[H^2]$ is invalid here.

As a final remark, we also point out that dynamical (not anomalous) generation of supersymmetric Chern–Simons terms exists in $D = 3$. For instance, in $D = 3$ supersymmetric nonlinear sigma models the latter are generated through a spontaneous breakdown of parity due to dynamical mass generation for the scalar superfield Φ [20, 7]. In the $N = 2$ formulation of these models [21], the mass generation manifests itself as a dynamical generation of the central charge (for the $D = 2$ case, see [22]).

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References

1. Redlich, A. N., *Phys. Rev.* **D29**, 2366 (1984);
   Jackiw, R., in R. Stora and B. S. de Witt (eds.), *Relativity, Groups and Topology II*, North-Holland, Amsterdam, 1984.
2. Alvarez-Gaumé, L. and Witten, E., *Nucl. Phys.* **B234**, 269 (1984).
3. Niemi, A. J. and Semenoff, G. W., *Phys. Rev. Lett.* **51**, 2077 (1983).
4. Jackiw, R., *Phys. Rev.* **D29**, 2375 (1984);
   Hughes, R., *Phys. Lett.* **148B**, 215 (1984).
5. Friedman, M., Sokoloff, J., Srivastava, Y., and Widom, A., *Phys. Rev. Lett.* **52**, 1587 (1984);
   Ishikawa, K., *Phys. Rev. Lett.* **53**, 1615 (1984).
6. Lott, J., *Phys. Lett.* **145B**, 179 (1984);
   Alvarez-Gaumé, L., Della Pietra, S., and Moore, G., *Ann. Phys.* (1985) (to appear).
7. Nissimov, E. R. and Pacheva, S. J., *Phys. Lett.* **146B**, 227 (1984).
8. Nissimov, E. R. and Pacheva, S. J., *Phys. Lett.* **157**, 407 (1985); in H. Doebner and T. D. Palev (eds.), *Proc. XIII Int. Conf. on Diff. Geom. Methods in Theor. Physics* (August 1984, Shumen, Bulg.), World Sci. (to be published);
   Egorian, E. S., Nissimov, E. R., and Pacheva, S. J., Sofia preprint INRNE-Feb-1985 (subm. to *Theor. Math. Phys.*).
9. Adler, S., in S. Deser *et al.* (eds.), *Lectures on Elementary Particles and Quantum Field Theory*, MIT Press, Cambridge MA, 1970;
   Jackiw, R., in *Lectures on Current Algebra and Its Applications*, Princeton Univ. Press, 1972.
10. Nissimov, E. R. and Pacheva, S. J., *.Phys. Lett.* **155B**, 76 (1985).
11. Honerkamp, J., Krause, F., Scheunert, M., and Schlindwein, M., *Nucl. Phys.* **B95**, 397 (1975);
    Nielsen, N. K., *Nucl. Phys.* **B244**, 499 (1984).
12. Atiyah, M. F., Patodi, V. K., and Singer, I. M., *Math. Proc. Camb. Phil. Soc.* **77**, 43 (1975); **78**, 405 (1975); **79**, 71 (1976).
13. James Gates, S., Grisaru, M., Rocek, M., and Siegel, W., *Superspace*, Benjamin Cummings, Reading, Mass., 1983.
14. Polyakov, A. M., unpublished (quoted in [8]).
15. Faddeev, L. D., Lecture notes, Leningrad Univ. (1975/6);
    Hawking, S. W., *Commun. Math. Phys.* **55**, 153 (1977).
16. Seeley, R., *Proc. Symp. Pure Math.* **10** (AMS **288**) (1967);
    Atiyah, M. F., Bott, R., and Patodi, V. K., *Invent. Math.* **19**, 279 (1973).
17. Treves, F., *Introduction to Pseudodifferential and Fourier Integral Operators*, Plenum Press, 1982.
18. Deser, S., Jackiw, R., and Templeton, S., *Ann. Phys.* **140**, 372 (1982).
19. Aragone, C., *Phys. Lett.* **131B**, 69 (1983).
20. Nissimov, E. R. and Pacheva, S. J., *Lett. Math. Phys.* **5**, 67 (1981); **6**, 361 (1982).
21. D'Adda, A., Lüscher, M., and Di Vecchia, P., *Nucl. Phys.* **B152**, 125 (1979).
22. Krivoshchekov, V. K. and Medvedev, P. B., *Phys. Lett.* **149B**, 138 (1984).